Life insurance portfolio aggregation

Is it optimal to group policyholders by age, gender, and seniority for BEL computations based on model points?

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A bit of Context

Solvency II

- EU directive that codifies and harmonizes the EU prudential framework
  - Amount of capital to reduce the risk of insolvency
- Enforcement on the 1\textsuperscript{st} of January, 2016

Best Estimate Liability computations via stochastic ALM
Cash-Flows Projection Model

- Capture the strong interaction between asset and liability
- Take into account the time value of options and guarantees

The Running Time issue

- Monte-Carlo simulations + Policy-by-policy approach
- AXA France participating contracts portfolio ⇒ 3 millions
  - Cumbersome volume of computations
Executive Summary

What is a model point?

A two-step procedure

- Clustering algorithms used in the field data analysis group policies to yield the clustered portfolio
- An aggregation procedure builds a representative contract for each group and yields the aggregated portfolio
- Aggregated portfolio of 4000 model points associated to relative error on the BEL of 0.05%
- Official AXA France Model Point building process since the closing of 2013
The Present Surrender Value of a participating contract

- \( \{r_a(t)\}_{t \geq 0} \) and \( \{r_d(t)\}_{t \geq 0} \) are stochastic processes governed by a probability measure \( \mathbb{P}_f \) that model respectively the accumulation and discounting rate.

- Let \( \mathbf{F} \) be a financial scenario drawn from \( \mathbb{P}_f \).

### The Surrender Value

\[
SV^F(t) = SV(0) \times \exp \left( \int_0^t r_a(s) ds \right),
\]

### The Present Surrender Value

\[
PSV^F(t) = SV^F(t) \times \exp \left( -\int_0^t r_d(s) ds \right),
\]
The Surrender Probability

Let $\tau|F$ be a continuous random variable that models the time of early surrender due to

- Death $\Rightarrow$ Age and Gender of the policyholder
- Lapse $\Rightarrow$ Seniority of the contract and financial scenario $F$

Let $T$ be the term of the contract or end of the horizon of projection.

- The actual surrender time is $\tau|F \wedge T = \min(\tau|F, T)$ with probability measure

$$
dP_{\tau|F \wedge T}(t) = f_{\tau|F}(t)d\lambda(t) + F_{\tau}(T)\delta_T(t)
$$
Theoretical Best Estimate Liability

Mean of the present value of the future exiting Cash-Flows weighted by their probability of occurrence

Given a Financial Scenario $F$

$$BEL^F(0, T) = \mathbb{E}[PSV(\tau|F \land T)] = \int_{0}^{T} SV(0) \times \exp \left[ \int_{0}^{t} (r_a(s) - r_d(s)) ds \right] d\mathbb{P}_{\tau|F \land T}(t)$$

Over a set of Financial Scenarios $(F_1, \ldots, F_N)$

$$BEL(0, T) = \frac{1}{N} \sum_{i=1}^{N} BEL^{F_i}(0, T)$$
Best Estimate Liability For Practitionners

Approximation through a discretization of time

\[ \text{BEL}^F(0, T) \approx \left[ \sum_{t=0}^{T-1} p(t, t+1) \prod_{k=0}^{t} \frac{1 + r_a(k, k+1)}{1 + r_d(k, k+1)} \right] \text{SV}(0) \]

\[ + \left[ p(T) \prod_{k=0}^{T-1} \frac{1 + r_a(k, k+1)}{1 + r_d(k, k+1)} \right] \text{SV}(0), \]

where

- Time step equal to one year
- Horizon of projection equal to 30 years
- \( p(t, t + 1) \) is the probability of surrender between year \( t \) and \( t + 1 \)
- \( p(T) \) is the probability to reach the end of projection year
- \( r_a(k, k + 1) \) and \( r_d(k, k + 1) \) denote the accumulation and discounting forward rate
Aggregation Philosophy

BEL Computation of a portfolio \((C_1, C_2)\)

- Let \(C_1\) and \(C_2\) have identical probabilities of surrender over the years
- \(SV_{MP}(0) = SV_{C_1}(0) + SV_{C_2}(0)\)

Then

\[
BEL_{MP}^F(0, T) = \sum_{i=1}^{2} BEL_{C_i}^F(0, T).
\]

- Exact valuation of the BEL of the portfolio \((C_1, C_2)\)

Getting as close as possible to this additivity property sounds like a good idea...
First Aggregation

Aggregation of contracts having

- Identical probabilities of surrender
- Identical ALM Group defined by features such as
  - Product Line
  - Benefit sharing features
  - Technical rate
  - ...

<table>
<thead>
<tr>
<th>Initial Portfolio</th>
<th>Portfolio After First Aggregation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maille ALM</td>
<td>Probability of Surrender</td>
</tr>
<tr>
<td>Grp_ALM_1</td>
<td>Vector_1</td>
</tr>
<tr>
<td>Grp_ALM_1</td>
<td>Vector_2</td>
</tr>
<tr>
<td>Grp_ALM_1</td>
<td>Vector_3</td>
</tr>
<tr>
<td>Grp_ALM_1</td>
<td>Vector_2</td>
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<tr>
<td>Grp_ALM_1</td>
<td>Vector_2</td>
</tr>
<tr>
<td>Grp_ALM_2</td>
<td>Vector_4</td>
</tr>
<tr>
<td>Grp_ALM_2</td>
<td>Vector_4</td>
</tr>
<tr>
<td>Grp_ALM_2</td>
<td>Vector_5</td>
</tr>
<tr>
<td>Grp_ALM_3</td>
<td>Vector_6</td>
</tr>
</tbody>
</table>
The Clustering Problem

Let

\[ \mathcal{P} = \{ \mathbf{C}_i \}_{i=1,...,n} \]

be a portfolio of contracts that belong to the same ALM Group

\[ \mathbf{C}_i = (p_i(0,1), p_i(1,2), ..., p_i(T - 1, T), p_i(T)) , \]

classified by their trajectory of surrender probabilities

▶ Giving up the financial dependency hypothesis
▶ Euclidean distance as dissimilarity measure
▶ AHC and K-MEANS Algorithm
▶ Weighting procedure based on the initial surrender value

\[ w_C = \frac{SV_C(0)}{\sum_{\mathbf{C} \in \mathcal{P}} SV_C(0)} , \]

▶ Similar to longitudinal data
A Meli-Melo of trajectories
Choice of the Clustering Method

Constraint on the number of Model Points

- Allocation of a number of model points to each ALM Group with respect to their mathematical reserves
- K-Means algorithm is better suited
  - The number of clusters is a parameter
- The random initialization is problematic
  - AHC to determine the initial centroid

Idea: Number of model points ⇒ Compromise between heterogeneity and mathematical reserve of the ALM Group
Combination of AHC and K-Means Then BOOM!
The problem reduces itself to assign the best characteristics to the MP

**The Simple Way**

- Weighted mean of the policyholder characteristics within the group

**A Trickier One**

- Generate every possible probability trajectories
- Compute the barycenter in each group
- Assign to the model point the characteristics leading to the trajectory which is closest to the barycenter
Overview of the Aggregation Process

- **Initial Ptf 4m**
  - **First Aggregation**
  - **Aggregated Ptf #1 72 000**
  - **Group ALM**
  - **Aggregated Ptf #2 4 000**
  - **Concatenation**

- **Sub Ptf #1**
- **Sub Ptf #n**

- **Clustering**
- **Aggregation**
Backtesting: Criteria and Figures

- $\mathcal{P}F_1$ denotes the aggregated portfolio after first aggregation
- $\mathcal{P}F_2$ denotes the final aggregated portfolio with the barycenter method
- The relative error on the BEL is defined as
  \[
  \frac{BEL(\mathcal{P}F_2) - BEL(\mathcal{P}F_1)}{BEL(\mathcal{P}F_1)}
  \]
- The compression rate is defined as
  \[
  \frac{\text{Card}(\mathcal{P}F_2) - \text{Card}(\mathcal{P}F_1)}{\text{Card}(\mathcal{P}F_1)}
  \]

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Number of Contracts</th>
<th>BEL (millions of euros)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{P}F_1$</td>
<td>72 000</td>
<td>72 336</td>
</tr>
<tr>
<td>$\mathcal{P}F_2$</td>
<td>3 753</td>
<td>72 371</td>
</tr>
</tbody>
</table>

- A relative error of 0.0485% equivalent to 35 millions of euros
- A compression rate of $-95\%$ VS $\mathcal{P}F_1$ and $-99.9\%$ VS policy-by-policy
Global Error over the years of projection

Evolution of the mathematical reserve and the net profit over the years of projection

The chronicles associated to the two portfolios are overlapping during the projection.
Relative Error on the BEL by Product Lines

99.09% of the Initial Surrender Value is associated to less than 0.5% of error.
Conclusion and Perspectives

Conclusion

▶ The aggregation procedure for participating contracts portfolios is very efficient
  ← Easy to implement
  ← Theoretically based and efficient in practice

▶ The aggregation procedure plays a key role within the valuation process of AXA France as
  ▶ It enables to do a full ALM valuation
  ▶ It meets the expectations of the regulators

There are Rooms for Further Improvements

▶ Try dissimilarity measures better suited to the problem than the euclidean distance
▶ Link the level of error to the number of Model Points
▶ Find a compromise between heterogeneity and mathematical reserve to allocate the number of model points
On the Optimal Number of Clusters

Weigted Within-Cluster Inertia

Calinski-Harabasz Index

Krzanowski-Lai Index

Silhouette Index
Inertia = Within-Cluster Inertia + Between-Cluster Inertia
K-Means Algorithm

**Step 1** Set the number of clusters
**Step 2** Random initialization of the centroids
**Step 3** Each individual is assigned to the closest centroid
**Step 4** Computations of new centroids
**Step 5** Repeat step 3 and 4 until convergence
Ascending Hierarchical Clustering Algorithm

Step 1  Group the two policies that minimize the increase of the Within-Cluster Inertia and replace them with the barycenter

Step 2  Repeat step 1 until only one group remains

Step 3  Cut the tree