Life insurance portfolio aggregation

Is it optimal to group policyholders by age, gender, and seniority for BEL computations based on model points?

Pierre-Olivier Goffard

Université Libre de Bruxelles
pierre-olivier.goffard@ulb.ac.be

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A bit of Context

Solvency II

- EU directive that codifies and harmonizes the EU prudential framework
  - $\hookrightarrow$ Amount of capital to reduce the risk of insolvency
- Enforcement on the 1^{st} of January, 2016

Best Estimate Liability computations via stochastic ALM Cash-Flows Projection Model

- Capture the strong interaction between asset and liability
- Take into account the time value of options and guarantees

The Running Time issue

- Monte-Carlo simulations + Policy-by-policy approach
- AXA France participating contracts portfolio $\Rightarrow$ 3 millions
  - $\hookrightarrow$ Cumbersome volume of computations

What is a model point?

A two-step procedure

- Clustering algorithms used in the field data analysis group policies to yield the clustered portofolio
- An aggreation procedure build a representative contract for each group and yield the aggregated portfolio
- Aggregated portfolio of **4000** model points associated to relative error on the BEL of **0.05%**
- Official AXA France Model Point building process since the closing of 2013

The Present Surrender Value of a participating contract

- $\{r_a(t)\}_{t\geq 0}$ and $\{r_d(t)\}_{t\geq 0}$ are stochastic processes governed by a probability measure $\mathbb{P}_f$ that model respectively the accumulation and discounting rate
- Let $\mathbf{F}$ be a financial scenario drawn from $\mathbb{P}_f$

The Surrender Value

$$SV^{\mathbf{F}}(t) = SV(0) \times exp\left(\int_0^t r_a(s)\mathrm{d}s\right),$$

The Present Surrender Value

$$PSV^{\mathbf{F}}(t) = SV^{\mathbf{F}}(t) \times exp\left(-\int_0^t r_d(s)ds\right),$$

The Surrender Probability

Let $\tau|\mathbf{F}$ be a continuous random variable that models the time of early surrender due to

- Death $\Rightarrow$ Age and Gender of the policyholder
- Lapse $\Rightarrow$ Seniority of the contract and financial scenario $\mathbf{F}$

Let T be the term of the contract or end of the horizon of projection.

- The actual surrender time is $\tau|\mathbf{F} \wedge T = \min(\tau|\mathbf{F}, T)$ with probability measure

$$\mathrm{d}\mathbb{P}_{\tau|\mathbf{F}\wedge T}(t) = f_{\tau|\mathbf{F}}(t)\mathrm{d}\lambda(t) + \overline{F_\tau}(T)\delta_T(t)$$

Theoretical Best Estimate Liability

Mean of the present value of the future exiting Cash-Flows weighted by their probability of occurence

Given a Financial Scenario $\mathbf{F}$

$$\begin{aligned} BEL^{\mathbf{F}}(0,T) &= \mathbb{E}\left[PSV\left(\tau|\mathbf{F}\wedge T\right)\right] \\ &= \int_0^T SV(0)\times \exp\left[\int_0^t (r_a(s)-r_d(s))ds\right] \mathrm{d}\mathbb{P}_{\tau|\mathbf{F}\wedge T}(t) \end{aligned}$$

Over a set of Financial Scenarios $(\mathbf{F}_1,\ldots,\mathbf{F}_N)$

$$BEL(0,T)=\frac{1}{N}\sum_{i=1}^{N} BEL^{\mathbf{F}_i}(0,T)$$

Approximation through a discretization of time

$$\begin{aligned} BEL^{\mathbf{F}}(0,T) &\approx \left[\sum_{t=0}^{T-1} p(t,t+1) \prod_{k=0}^{t} \frac{1+r_a(k,k+1)}{1+r_d(k,k+1)}\right] SV(0) \\ &+ \left[p(T) \prod_{k=0}^{T-1} \frac{1+r_a(k,k+1)}{1+r_d(k,k+1)}\right] SV(0), \end{aligned}$$

where

- Time step equal to one year
- Horizon of projection equal to 30 years
- $p(t,t+1)$ is the probability of surrender between year t and $t+1$
- $p(T)$ is the probability to reach the end of projection year
- $r_a(k,k+1)$ and $r_d(k,k+1)$ denote the accumulation and discounting forward rate

Aggregation Philosophy

BEL Computation of a portfolio (C_1, C_2)

- Let C_1 and C_2 have identical probabilities of surrender over the years
- $SV_{MP}(0) = SV_{C_1}(0) + SV_{C_2}(0)$

Then

$$BEL^{\mathbf{F}}_{MP}(0, T) = \sum_{i=1}^{2} BEL^{\mathbf{F}}_{C_i}(0, T).$$

- Exact valuation of the BEL of the portfolio (C_1, C_2)

Getting as close as possible to this additivity property sounds like a good idea...

First Aggregation

Aggregation of contracts having

- Identical probabilities of surrender
- Identical ALM Group defined by features such as
  - Product Line
  - Benefit sharing features
  - Technical rate
  - ...

Initial Portfolio		
Maille ALM	Probability of Surrender	Initial Surrender Value
Grp_ALM_1	Vector_1	100
Grp_ALM_1	Vector_2	200
Grp_ALM_1	Vector_3	50
Grp_ALM_1	Vector_2	80
Grp_ALM_1	Vector_2	3000
Grp_ALM_2	Vector_4	5000
Grp_ALM_2	Vector_4	200
Grp_ALM_2	Vector_5	40000
Grp_ALM_3	Vector_6	100
Grp_ALM_3	Vector_6	700
Grp_ALM_3	Vector_6	9000

Portfolio After First Aggregation		
Maille ALM	Probability of Surrender	Initial Surrender Value
Grp_ALM_1	Vector_1	100
Grp_ALM_1	Vector_2	3280
Grp_ALM_1	Vector_3	50
Grp_ALM_2	Vector_4	5200
Grp_ALM_2	Vector_5	40000
Grp_ALM_3	Vector_6	10250

The Clustering Problem

Let

$$\mathcal{P} = \{\mathbf{C}_i\}_{i \in 1,\dots,n}$$

be a portfolio of contracts that belong to the same ALM Group

$$\mathbf{C}_i = (p_i(0,1), p_i(1,2), \dots, p_i(T-1,T), p_i(T)),$$

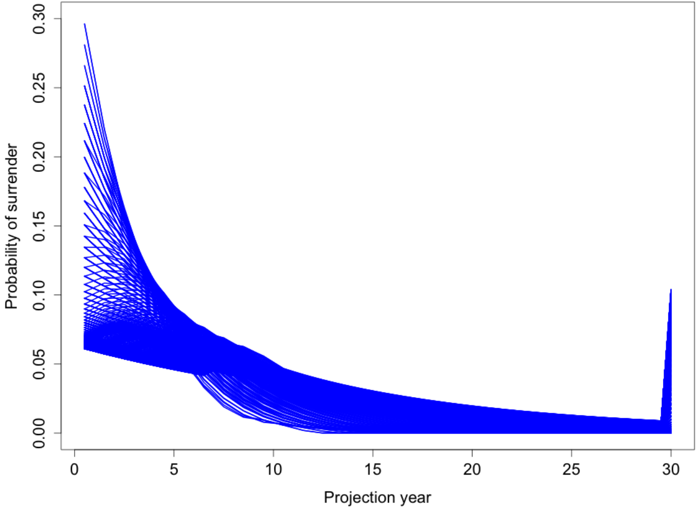
characterized by their trajectory of surrender probabilities

- Giving up the financial dependency hypothesis
- Euclidean distance as dissimilarity measure
- AHC and K-MEANS Algorithm
- Weighting procedure based on the initial surrender value

$$w_{\mathbf{C}} = \frac{SV_{\mathbf{C}}(0)}{\sum_{\mathbf{C} \in \mathcal{P}}^{n} SV_{\mathbf{C}}(0)},$$

- Similar to longitudinal data

A *Meli-Melo* of trajectories



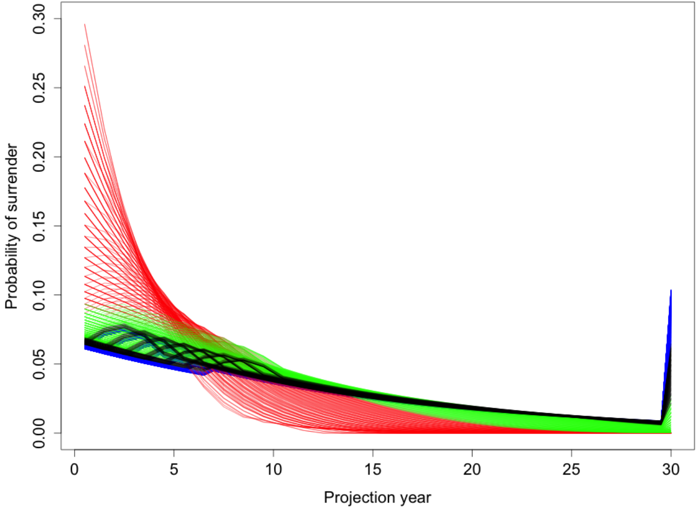
Choice of the Clustering Method

Constraint on the number of Model Points

- Allocation of a number of model points to each ALM Group with respect to their mathematical reserves
- K-Means algorithm is better suited
  - $\hookrightarrow$ The number of clusters is a parameter
- The random initialization is problematic
  - $\hookrightarrow$ AHC to determine the initial centroid

Idea Number of model points $\Rightarrow$ Compromise between heterogeneity and mathematical reserve of the ALM Group

Combination of AHC and K-Means Then BOOM!



The Aggregation Step: Two Ways

The problem reduces itself to assign the best characteristics to the MP

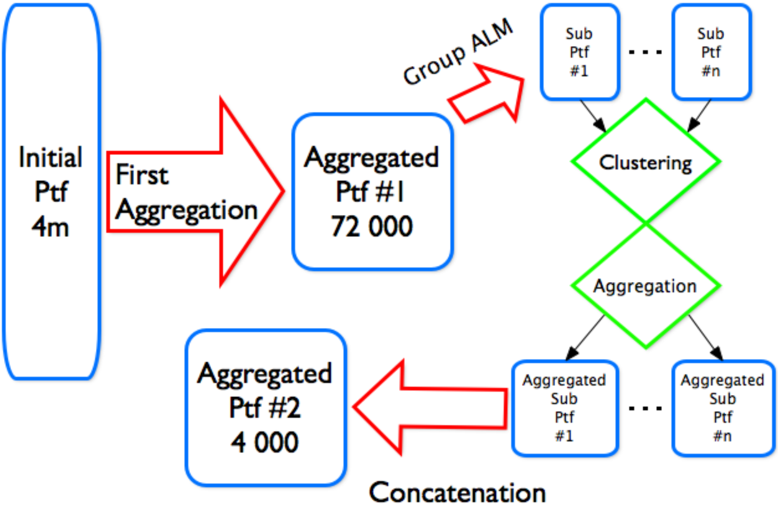
The Simple Way

- Weighted mean of the policyholder characteristics within the group

A Trickier One

- Generate every possible probability trajectories
- Compute the barycenter in each group
- Assign to the model point the characteristics leading to the trajectory which is closest to the barycenter

Overview of the Aggregation Process



Backtesting: Criteria and Figures

- $\mathcal{PF}_1$ denotes the aggregated portfolio after first aggregation
- $\mathcal{PF}_2$ denotes the final aggregated portfolio with the barycenter method
- The relative error on the BEL is defined as

$$\frac{BEL\left(\mathcal{PF}_2\right) - BEL\left(\mathcal{PF}_1\right)}{BEL\left(\mathcal{PF}_1\right)}$$

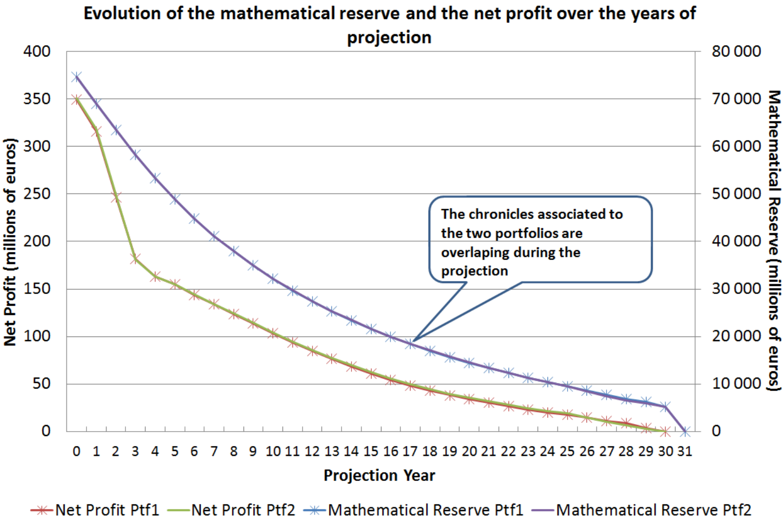
- The compression rate is defined as

$$\frac{Card\left(\mathcal{PF}_2\right) - Card\left(\mathcal{PF}_1\right)}{Card\left(\mathcal{PF}_1\right)}$$

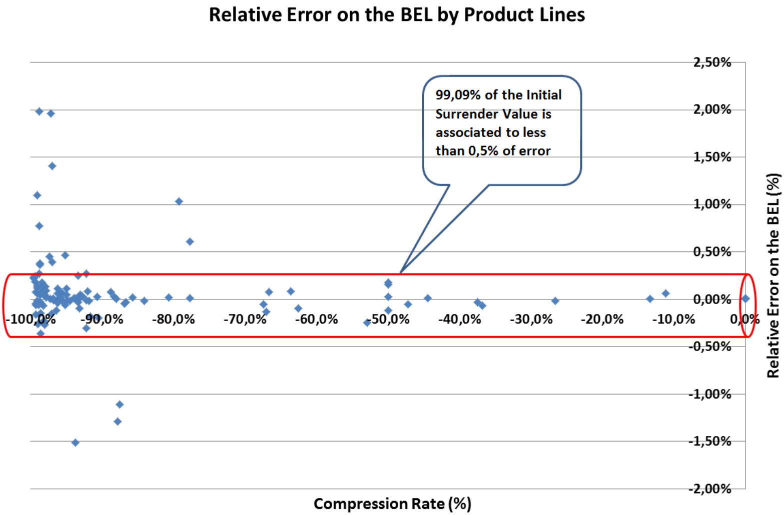
Portfolio	Number of Contracts	BEL (millions of euros)
$\mathcal{PF}_1$	72 000	72 336
$\mathcal{PF}_2$	3 753	72 371

- A relative error of **0.0485%** equivalent to **35** millions of euros
- A compression rate of $-$**95%** VS $\mathcal{PF}_1$ and $-$**99.9%** VS policy-by-policy

Global Error over the years of projection



Compression Rate VS Relative Error Product-by-Product



Conclusion and Perspectives

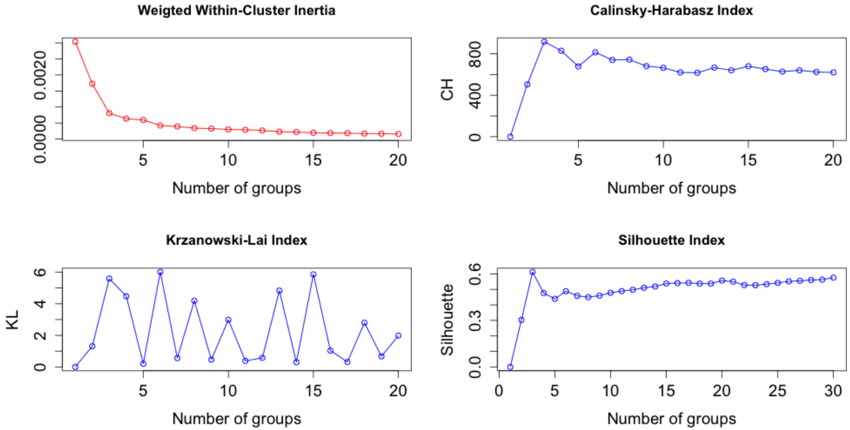
Conclusion

- The aggregation procedure for participating contracts portfolios is very efficient
  - ↪ Easy to implement
  - ↪ Theoretically based and efficient in practice
- The aggregation procedure plays a key role within the valuation process of AXA France as
  - It enables to do a full ALM valuation
  - It meets the expectations of the regulators

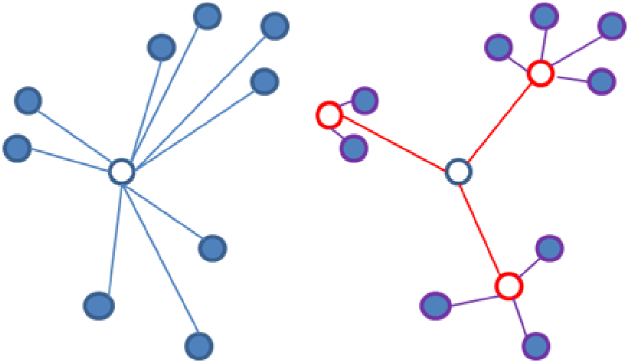
There are Rooms for Further Improvements

- Try dissimilarity measures better suited to the problem than the euclidean distance
- Link the level of error to the number of Model Points
- Find a compromise between heterogeneity and mathematical reserve to allocate the number of model points

On the Optimal Number of Clusters



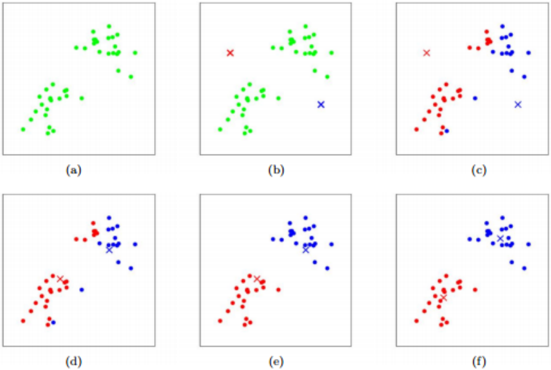
Clustering Philosophy



Inertia = Within-Cluster Inertia + Between-Cluster Inertia

K-Means Algorithm

Step 1 Set the number of clusters
Step 2 Random initialization of the centroids
Step 3 Each individual is assigned to the closest centroid
Step 4 Computations of new centroids
Step 5 Repeat step 3 and 4 until convergence



Ascending Hierarchical Clustering Algorithm

Step 1 Group the two policies that minimize the increase of the Within-Cluster Inertia and replace them with the barycenter

Step 2 Repeat step 1 until only one group remains

Step 3 Cut the tree