Stochastic Models for blockchain analysis

Introduction

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BFS summer school

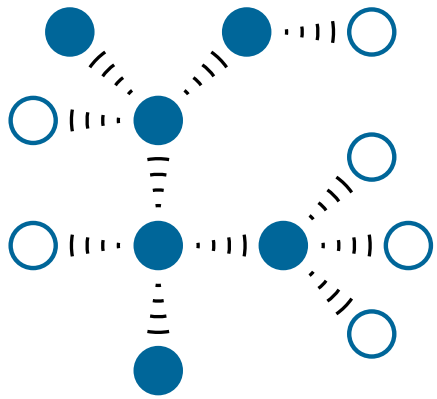
Mini course on the topic of blockchain at the 1^{st} Bachelier Finance Society Summer school.

- Link to the event `https://www.bachelierfinance.org/09-2021`
- Link to the course material `https://pierre-olivier.goffard.me/BLOCKASTICS/`

Blockchain

A data ledger made of a sequence of blocks maintained by a achieving consensus in a Peer-To-Peer network.

- Decentralized
- Public/private
- Permissionned/permissionless
- Immutable
- Incentive compatible



We will focus on public blockchain and their associated consensus protocol.

Blocks

A block contains

- block height/ID
- Time stamp
- hash of the block
- hash of the previous block
- Set of transactions (data stored in the blockchain)

```
Block Height: 0
Block Hash: a52bea61a9f4131588cc101e8e1c731fa9f69f16934c5ab3a05a2134a42c13e0
Time:2021-07-12 10:03:04.812744
Block data: [{'sender': 'Coinbase', 'recipient': 'Satoshi', 'amount': 100, 'fee': 1}]
Mined: False
Previous block hash: 0
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```

Cryptographic Hash function

A function that maps data of arbitratry size (message) to a bit array of fixed size (hash value)

$$h : \{0,1\}^* \mapsto \{0,1\}^d.$$

A good hash function is

- deterministic
- quick to compute
- One way
  - ↪ For a given hash value $\overline{h}$ it is hard to find a message m such that

$$h(m) = \overline{h}$$

- Colision resistant
  - ↪ Impossible to find m_1 and m_2 such that

$$h(m_1) = h(m_2)$$

- Chaotic

$$m_1 \approx m_2 \Rightarrow h(m_1) \neq h(m_2)$$

Consensus protocols

The mechanism to make all the nodes agree on a common data history.

The three dimensions of blockchain systems analysis

1. Efficiency (Queueing theory)
   - Throughputs
   - Transaction confirmation time
2. Decentralization (Entropy)
   - Fair distribution of the accounting right
3. Security (Insurance Risk Theory)
   - Resistance to attacks

X. Fu, H. Wang, and P. Shi, "A survey of blockchain consensus algorithms : mechanism, design and applications," *Science China Information Sciences*, vol. 64, nov 2020.

Proof of Work

The nodes compete to solve a cryptographic problem by brute force search.

PoW

1. Draw a random number (nonce)

$$X \sim \{1, \ldots, 2^{32}\}.$$

2. While $X > L$, where L is the target then try again

Nodes are chosen according to their computing power

S. Nakamoto, "Bitcoin : A peer-to-peer electronic cash system." Available at https://bitcoin.org/bitcoin.pdf, 2008.

Proof of Stake

PoW is slow and ressource consuming. Let $\{1,\ldots,N\}$ be a set of miner and $\{\pi_1,\ldots,\pi_N\}$ be their share of cryptocoins.

PoS

Node $i \in \{1,\ldots,N\}$ is selected with probability π_i to append the next block

Nodes are chosen according to what they own.

- Nothing at stake problem
- Rich gets richer ? (To be discussed later on)

F. Saleh, "Blockchain without waste : Proof-of-stake," *The Review of Financial Studies*, vol. 34, pp. 1156–1190, jul 2020.

Applications of blockchain : Cryptocurrency

S. Nakamoto, “Bitcoin : A peer-to-peer electronic cash system.” Available at https://bitcoin.org/bitcoin.pdf, 2008.

- Transaction anonymity
- Banking and reliable currency in certain regions of the world
- Money Transfer worldwide (at low fare)
- No need for a thrusted third party

How does it work ?

1. No central authority (Decentralized network)
2. Ledger to record all the transactions and coin ownership (blockchain)
3. A coin generation process (block finding reward)
   ↪ Incentive to the full nodes
4. Ownership can be proved cryptographically (wallet associated to a public/private key)
5. Transactions can be issued by an entity proving ownership of the cryptographic unit (through the private key)
6. The system cannot process more than one transaction associated to the same cryptographic unit (double spending)

J. Lansky, "Possible state approaches to cryptocurrencies," *Journal of Systems Integration*, vol. 9, pp. 19–31, jan 2018.

Cryptocurrency implementation

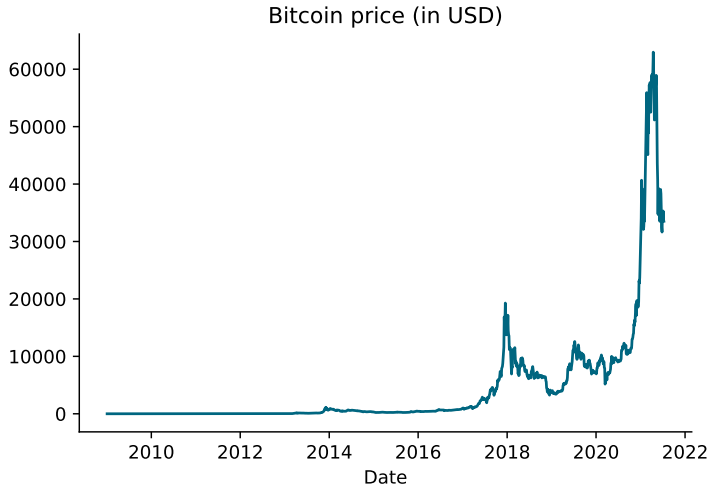
Blockchain parameters

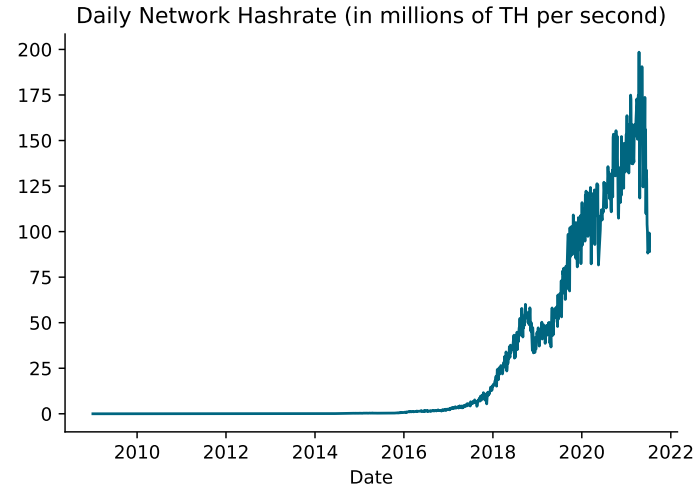
- Consensus protocol (PoW or PoS)
  - ↪ Hash function (SHA-256 for Bitcoin and scrypt for LiteCoin)
  - ↪ Hybrid PoW/PoS (PeerCoin)
- Block generation time
  - ↪ every 10 minutes for Bitcoin
  - ↪ every 12 sec for Ethereum
- Block finding reward
  - ↪ Halved every 210,000 blocks in Bitcoin. It started at 50 BTC, is now 6.25 BTC `https://www.bitcoinblockhalf.com/`
- Total coin supply
  - ↪ 21,000,000 in total for Bitcoin
- Transaction fees
  - ↪ GAS in Ethereum

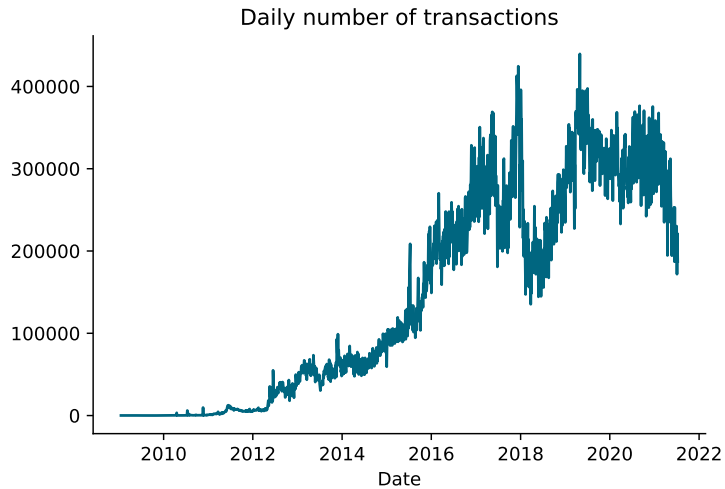
These choices lead to the creation of multiple cryptocurrencies

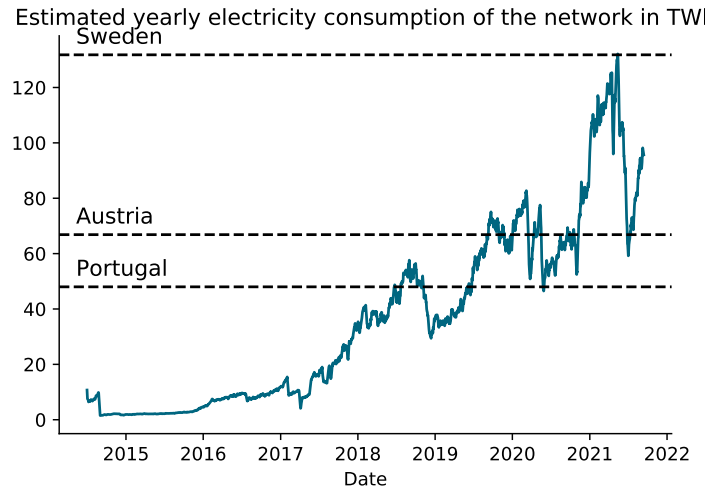
Examples

Bitcoin and AltCoins (Ethereum, LiteCoin, DogeCoin, Ripple...), see `https://en.wikipedia.org/wiki/List_of_cryptocurrencies`









. Sources : https://www.blockchain.com/ & https://cbeci.org/

Decentralized application

The network provide ressources such as

- storage
- computing power

through a smart contract on the ethereum blockchain.

GOLEM (`https://www.golem.network/`)

Build a network of idle computers to do paralell computing.

Utility tokens are used to access the service and provision the network ressources.

Equation of Exchange (Fisher 1911)

$$MV = PQ$$

Decentralized finance

DEFI creates new financial architecture

+ Non custodial
+ Anonymous
+ Permisionless
+ openly auditable

- Unregulated
- Tax evasion
- Fraud
- Money laundering

Extends the Bitcoin promises to more complex financial operations

- Collateralized lending
- Decentralized Exchange Platform
- Tokenized assets
- Fundraising vehicle (ICO, STO, ...)

S. M. Werner, D. Perez, L. Gudgeon, A. Klages-Mundt, D. Harz, and W. J. Knottenbelt, "Sok : Decentralized finance (defi)," 2021.

Tokenized real-world assets

Tokenized version of a real-world, physical asset

- Increases the liquidity of certain type of assets
- Make certain classes of assets available to the many
- Can be used as store of value or collateral

These token can be backed by

- fiat currency ⇒ stablecoin
- commodities like gold `https://ekon.gold/`
- stocks (security token) that includes voting right and profit sharing mechanism
- Art
- Digital art (Non Fungible tokens on the Ethereum blockchain)

Central authority

This requires a custodian to ensure that the tokens are actually backed by these off-chain assets (except for NFTs).

OECD, "The tokenisation of assets and potential implications for financial markets," tech. rep., 2020.

Valuation models

- Cryptocurrencies are medium of exchange and may be priced via transaction cost model (Beaumol-Tobin and such)

W. J. Baumol, "The transactions demand for cash : An inventory theoretic approach," *The Quarterly Journal of Economics*, vol. 66, p. 545, nov 1952.

L. Schilling and H. Uhlig, "Some simple bitcoin economics," *Journal of Monetary Economics*, vol. 106, pp. 16–26, oct 2019.

- Tokenized asset depends on the real asset that backs the token

J. Hargrave, N. Sahdev, and O. Feldmeier, "How value is created in tokenized assets," in *Blockchain Economics : Implications of Distributed Ledgers*, pp. 125–143, WORLD SCIENTIFIC (EUROPE), jan 2019.

- Utility tokens

J. R. Gan, G. Tsoukalas, and S. Netessine, "Initial coin offerings, speculation, and asset tokenization," *Management Science*, vol. 67, pp. 914–931, feb 2021.

L. W. Cong, Y. Li, and N. Wang, "Tokenomics : Dynamic adoption and valuation," *The Review of Financial Studies*, vol. 34, pp. 1105–1155, aug 2020.

ICO tuning and timeline

1 ICO period

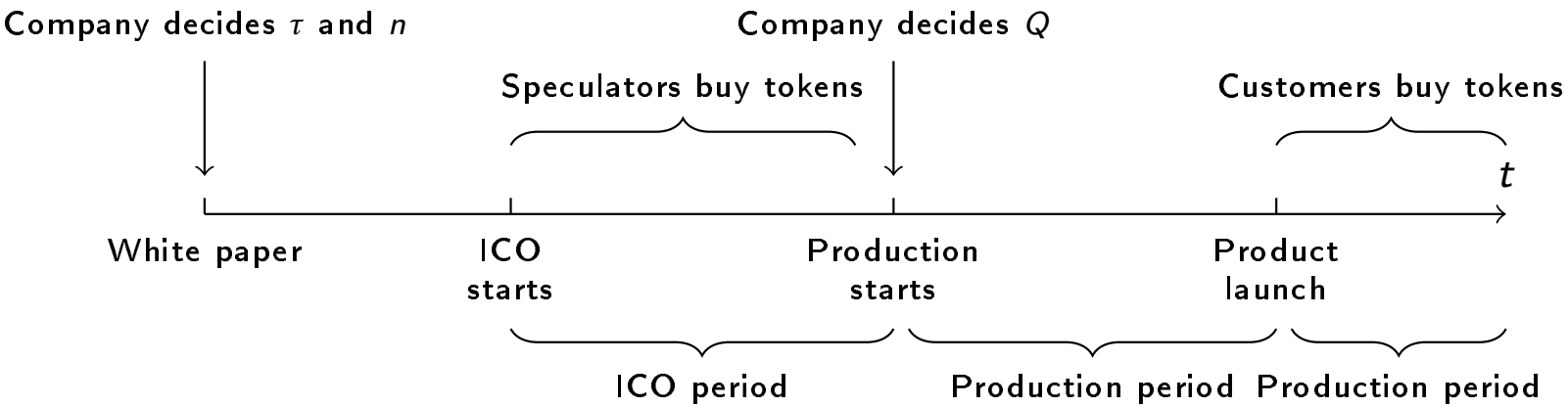
- The firm publishes a white paper and set
  - The token price τ
  - The total number of token m
  - The number of token issued to the investors during the ICO $n \leq m$.
- s among $z >> m$ investors buy token

2 Production period

- The firm uses the funds raised $s\tau$ to finance the production of Q units of goods

3 Market period

- Customers purchase token to meet their needs $D \sim F(.)$



J. R. Gan, G. Tsoukalas, and S. Netessine, "Initial coin offerings, speculation, and asset tokenization," *Management Science*, vol. 67, pp. 914–931, feb 2021.

Insights

Optimal number of tokens sold n^*

The more token the firm sells during the ICO

- The more money to invest in production
- The less tokens it has to sell in the secondary market
- The less "skin in the game"
- The less it wants to invest in production ex post

n^* resolves the trade off between money now and money later while controlling moral hazard.

Optimal token price τ^*

- Price too low : Not enough funds raised
- Price too high : not enough upside for investors

Gerry Tsoukalas talk at
`https://www.youtube.com/watch?v=E_NT4t4ws8U`

Decentralized insurance

Parametric insurance

Compensation if a measurable quantity reaches a threshold

- Example : Flight delay insurance
  - `https://etherscan.io/address/0xdc3d8fc2c41781b0259175bdc19516f7da11cba7`
- Use smart contract and off-chain data through oracles
- Transparent and automatic